

Engineering Report ER7024

March 1972

AD 742851

# FURTHER RESEARCH ON THE $I_p$ STATISTIC

By

R. Bruce Young

Prepared For

Infantry Team, Systems Performance and Concept Directorate  
Human Engineering Laboratory

USA Aberdeen Research and Development Center  
Aberdeen Proving Ground, Maryland 21005

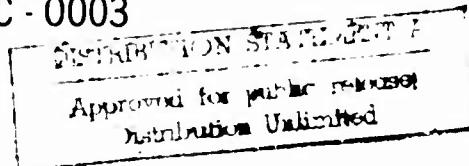
By

**AAI Corporation**

BALTIMORE, MARYLAND 21204

Under Contract No. DAAD05-72-C-0003

Reproduced by  
NATIONAL TECHNICAL  
INFORMATION SERVICE  
Springfield, Va. 22151



**DDC AVAILABILITY NOTICE**

**Qualified requestors may obtain copies of this report from DDC.**

**Destroy this report when no longer needed.  
Do not return it to the originator.**

**This document has been approved for public release and sale;  
its distribution is unlimited.**

**The findings in this report are not to be construed as an official  
Department of the Army position, unless so designated by other  
authorized documents issued and approved by the Department of the  
Army.**

ACCESSION for	
CFSTI	WHITE SECTION <input checked="" type="checkbox"/>
DOC	BUFF SECTION <input type="checkbox"/>
UNANNOUNCED	<input type="checkbox"/>
JUSTIFICATION	
BY	
DISTRIBUTION AVAILABILITY CODES	
DIST	AIR MAIL AND AIR SPECIAL
A1	

**UNCLASSIFIED**

Security Classification

**DOCUMENT CONTROL DATA - R & D**

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author)		2a. REPORT SECURITY CLASSIFICATION
AAI Corporation		2b. GROUP
3. REPORT TITLE		
Further Research on the $I_p$ Statistic		
4. DESCRIPTIVE NOTES (Type of report and inclusive dates)		
5. AUTHOR(S) (First name, middle initial, last name)		
R. Bruce Young		
6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
March 1972	40	1
8a. CONTRACT OR GRANT NO.	9a. ORIGINATOR'S REPORT NUMBER(S)	
DAAD05-72-C-0003	ER-7024	
8b. PROJECT NO.	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)	
8c.	9c.	
8d.	9d.	
10. DISTRIBUTION STATEMENT		
This document has been approved for public release and sale; its distribution is unlimited.		
11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY	
	Infantry Team, Systems Performance and Concept Directorate, Human Engineering Laboratory, USA Aberdeen Research & De- velopment Center, Aberdeen Proving Ground, Maryland	
13. ABSTRACT		
Computer simulations were used to determine the influence of projectile impact point assessment errors on the validity of the computation of the index of proximity statistic. The results of the study showed that measurement errors of the magnitude currently experienced in the field will not affect the validity of the computation. Subsequent simulations were used to demonstrate the response of the statistic to several types of impact patterns.		

**UNCLASSIFIED**

Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Fire suppression						
Effectiveness						
Tracer Ammunition						
Scoring technique						
Human Engineering						
Weapons Testing						
Computer Simulation						

**UNCLASSIFIED**

Security Classification

Engineering Report ER7024

March 1972

# FURTHER RESEARCH ON THE $I_p$ STATISTIC

By

R. Bruce Young

Prepared For

Infantry Team, Systems Performance and Concept Directorate  
Human Engineering Laboratory  
USA Aberdeen Research and Development Center  
Aberdeen Proving Ground, Maryland 21005

By

**AAI Corporation**

BALTIMORE, MARYLAND 21204

Under Contract No. DAAD05-72-C-0003

ABSTRACT

Computer simulations were used to determine the influence of projectile impact point measurement errors on the accuracy of a computed index of proximity statistic. The results of the study showed that measurement errors of the magnitude currently experienced in the field will not affect the accuracy of the statistic. Subsequent simulations were used to demonstrate the response of the statistic to several types of impact patterns.

TABLE OF CONTENTS

	<u>Page</u>
I. INTRODUCTION - - - - -	1
II. EXERCISES AND RESULTS - - - - -	3
III. COMPARISON WITH OTHER STATISTICAL MEASURES - - - - -	28
IV. CONCLUSIONS - - - - -	33

## I. INTRODUCTION

To process the data to be provided by miss-distance indicator (MDI) systems currently under development, a statistical procedure was developed for assessing and expressing in an index number the proximity to the target of impact points for serially fired small arms rounds. This index number is named the "index of proximity" and has the notation  $I_p$ . The index of proximity is intended as a supplement to other scoring measures such as hits/shots ratios. It is particularly intended for scoring groups of missions in which some shooters experience improving proximity to the target, but the actual number of hits on the target by all shooters is very low. Additional features of the  $I_p$  are that certain factors which are thought to contribute to suppressiveness have been included and the relative importance of these factors can be adjusted by the user through weighting factors. The index is presented in Appendix A.

In order to compute  $I_p$ , the impact point coordinates for each round must be known. Results of recent field tests have indicated that, in at least some instances, the instrumentation errors associated with measuring the impact points will be significant compared to the actual miss distances being measured. The primary intent of the effort described in this report was to determine whether these instrumentation errors would introduce statistically significant variations between the true and measured  $I_p$ 's for a given set of conditions.

In addition to determining the effect of measurement errors on the  $I_p$  statistic, the sensitivity of the index and its ability to discriminate between different types of shooting patterns were also examined. The

efficiency of the  $I_p$  as a measure of goodness was also compared with that of certain other statistics.

A Monte Carlo computer simulation was used in eight separate exercises to evaluate the effects of changes in the magnitude of the aim error, ballistic error, and measurement error on  $I_p$ . The basic approach in each exercise was to establish aim, ballistic<sup>1</sup>, and measurement errors in terms of linear standard deviations. A draw from the aim error distribution was added to a draw from the ballistic error distribution. The coordinates thus obtained were assumed to describe "actual" impact points. The same draw from the aim error distribution was next added to a draw from the measurement error distribution. The coordinates determined by this summation were assumed to describe "measured" impact points. This process was repeated for the nth round in the mission.  $I_p$ 's were then computed for the "actual" impact point coordinates and the "measured" impact point coordinates for each mission. This procedure was repeated for 50, 100, and 200 missions. Mean  $I_p$ 's were computed for these three mission levels. The difference between the mean indices for the "actual" and "measured" conditions at each of the three mission levels was tested for statistical significance at the .05 level. Further details of these computations are described in the discussions of the individual exercises. The machine computations were performed on the Ballistic Research Laboratories' Engineering and Scientific Computer (BRLESC) at Aberdeen Research and Development Center.

---

<sup>1</sup>The "ballistic" error as used in these exercises was a normally distributed error centered about the aim point for each shot. Thus, the effect simulated is that of any system variable which has those characteristics.

## II. EXERCISES AND RESULTS

### Exercise No. 1

The purpose of this exercise was to determine the sensitivity of  $I_p$  to variations in measurement error for combinations of aim and ballistic error. The following errors were assumed:

<u>Measurement Error S.D.</u>	<u>Ballistic Error S.D.</u>	<u>Aim Error S.D.</u>
0 mil	.3 mil	1 mil
.1 mil	.5 mil	3 mil
.3 mil	.7 mil	5 mil
.5 mil	.9 mil	10 mil
1.0 mil		

Each mission consisted of 5 rounds (or fewer if a hit occurred) fired at a target 18 inches wide by 36 inches high (roughly the size of the Army's standard E-type silhouette) located at a range of 500 meters.

All possible combinations of the above variables were used as inputs to the program.

The results of this exercise showed that measurement errors of the magnitude studied had no adverse affect on the computed  $I_p$  when summed over 50 missions or more. The computed values for the worst case (1 mil measurement error with 1 mil aim and .3 ballistic errors) were:

<u>Aim Error</u>	<u>Ballistic Error</u>	<u>Measurement Error</u>	<u>No Missions</u>	<u><math>I_p</math></u>
1	.3	0	50	.7266
1	0	1		.7380
1	.3	0	100	.7064
1	0	1		.7053
1	.3	0	200	.6898
1	0	1		.6922

The difference between the "actual"  $I_p$  and the "measured"  $I_p$  at each mission level is not significant at the .05 level. Also, the differences between the  $I_p$ 's for 50 missions and for 200 missions are not significant. This finding held for all of the error combinations studied. Therefore, if the errors and shot patterns actually experienced in the field are in fact well represented by normal distributions, 50 missions should be sufficient to establish  $I_p$  with a high degree of confidence.

Figure 1 shows the range of  $I_p$  that was computed for the aim and ballistic errors considered in this exercise.

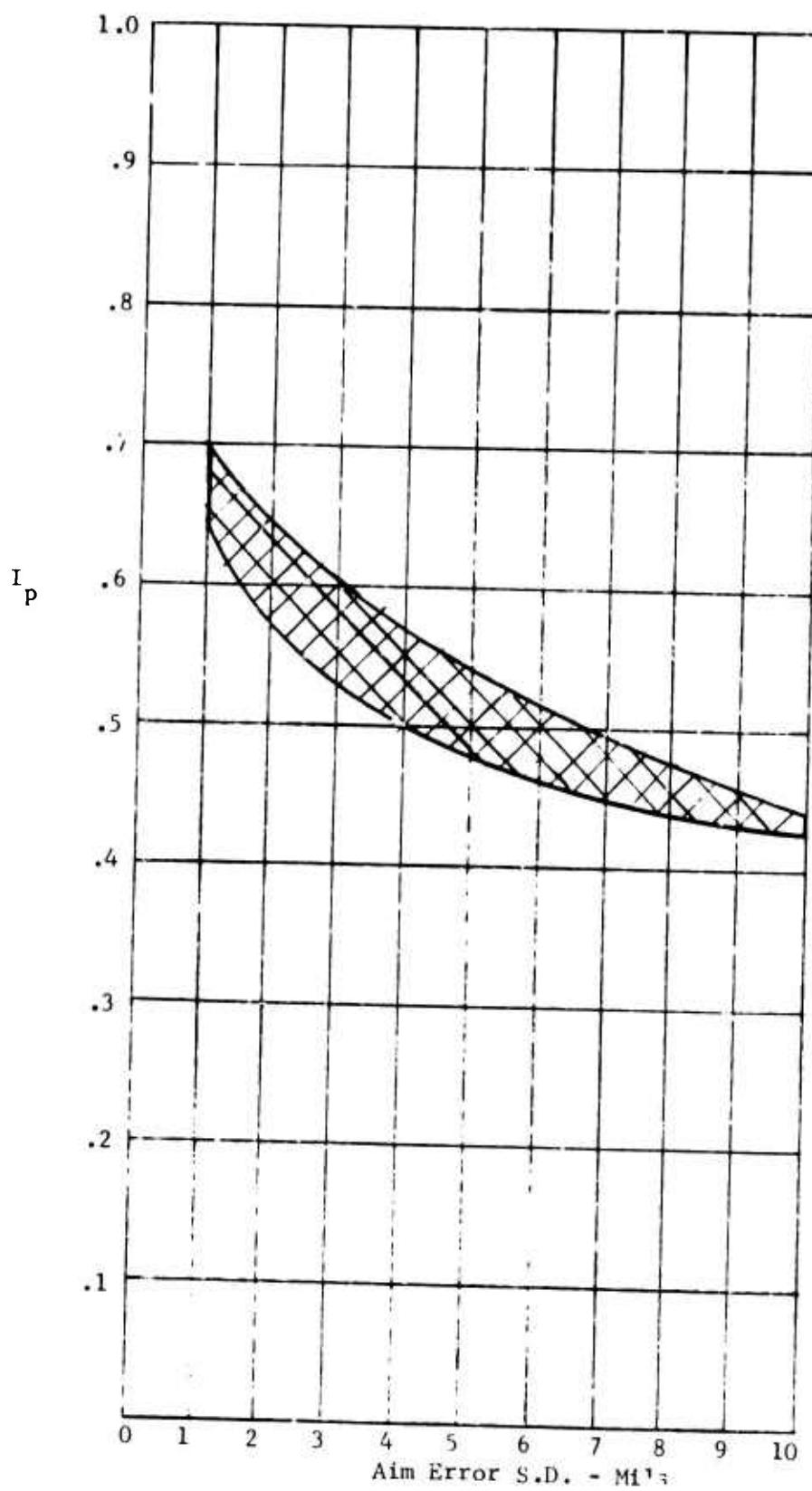


Figure 1.  $I_p$  as a Function of Aim Error for Ballistic Errors of .3 to .9 Mils at the 200 Mission Level.

### Exercise No. 2

For some measurement systems it may be difficult to detect, or eliminate, a bias error. That is, an error in the measurement which is constant in magnitude and direction. To study the effect of a bias measurement error on  $I_p$ , simulations were made for the following cases:

<u>Constant Measurement Error</u>	<u>Ballistic Error S.D.</u>	<u>Aim Error S.D.</u>
0 mil	.5 mil	1 mil
.2 mil		3 mils
.4 mil		
.6 mil		10 mils

All combinations of those errors were considered. The target was again 18 inches by 36 inches and located at a range of 500 meters. The mission size was 5 rounds.

The results of this exercise showed that the difference between the actual  $I_p$  and the  $I_p$  computed with the bias measurement error is not significant at the .05 level for sample sizes of 50, 100, and 200 missions.

On the basis of these first two exercises it is concluded that when the  $I_p$  is summed over 50 missions or more, variation in the  $I_p$  due to impact point measurement error is not significant.

Subsequent exercises described in this report were directed toward a study of the statistic itself. Hence, only "actual" impact points are considered.

Exercise No. 3

One function of the  $I_p$  is to measure improving proximity of rounds to the target. During the development of the statistic, it was reasoned that a first round hit left no room for improvement and hence was not scorable. A second round hit is perfect improvement and achieves a perfect  $I_p$  score<sup>1</sup> of 1.0. In an actual field experiment, the user will probably establish the mission size<sup>2</sup> before the test is started. If the target is large compared to the aim and ballistic errors, there may be a large number of first round hit missions; hence, a large number of missions which are not scorable with the  $I_p$ .

The purpose of this exercise was to determine whether, for a fixed number of missions, it was possible to get so many first round hits due to a favorable combination of aim error and target size that the  $I_p$  would not be responsive to actual improvements in aim error. The following variables were used:

<u>Ballistic Error S.D.</u>	<u>Aim Error S.D.</u>	<u>Target Size</u>	<u>Range<sup>3</sup> to Target</u>
.5 mil	1 mil	18"x36"	100 meters
	3 mils		300 meters
	5 mils		500 meters
	10 mils		

<sup>1</sup> Because of the weighting factor "low rounds," a second round hit produces a perfect score only if the first round impacted below the target.

<sup>2</sup> number of rounds to be fired in a single engagement.

<sup>3</sup> The effect of range was simulated by using three different-sized targets to produce the effect of the same size target at three different ranges. Neither projectile drop nor the presumed psychological effect upon the firer of the different target ranges was considered.

All possible combinations of these variables were tested. The mission levels were 50, 100 and 200.

It was found that in all cases the difference in  $I_p$  for any two aim errors for a given target range was statistically significant. That is, for 50 missions or more the  $I_p$  will discriminate between a 1 mil shooter and a 3 mil shooter firing at an 18 inch by 36 inch target located at a range of 100 meters.

Figure 2 shows the  $I_p$  as a function of aim error for the 3 target ranges at the 50 mission level.

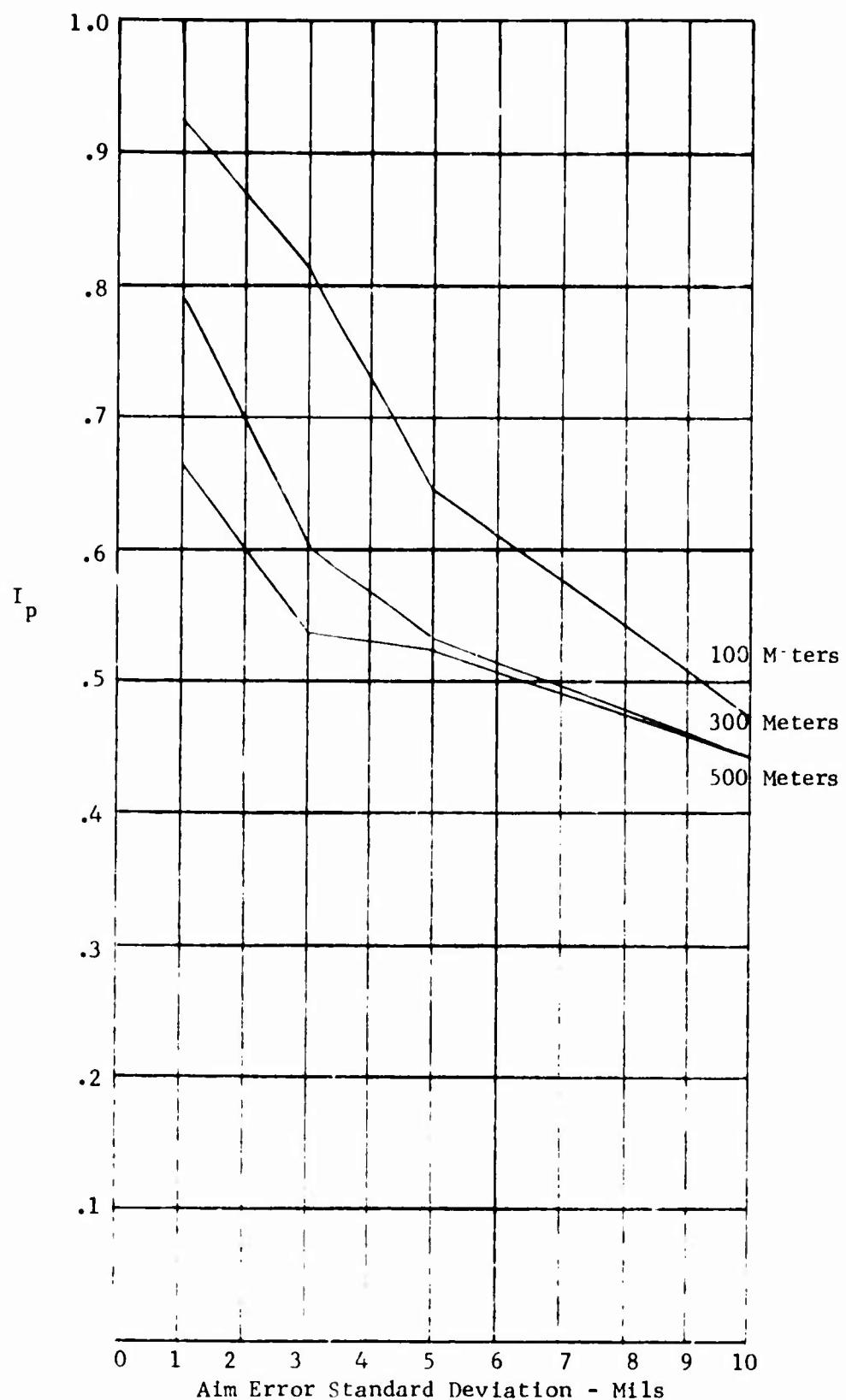


Figure 2.  $I_p$  as a Function of Aim Error for Three Target Ranges

Exercise No. 4

Short target exposure times may limit the number of rounds that can be fired in one mission. This exercise was designed to study the influence of number of rounds per mission on the value of  $I_p$ . All combinations of these values were studied:

<u>No. Shots Per Mission</u>	<u>Ballistic Error S.D.</u>	<u>Aim Error S.D.</u>
3	.5 mil	1 mil
4		3 mils
5		5 mils
		10 mils

The target size was again 18 inches by 36 inches, and the range was 500 meters. Mission levels were 50, 100, and 200.

A summary of the results is given in Table 1.

The differences between the  $I_p$ 's were significant at the .05 level in just two instances: 5 shot mission vs. 3 shot mission for 3 mil aim error at 100 mission level; and 3 shot mission vs. 4 shot mission for 10 mil aim error at 50 mission level. All other comparisons between  $I_p$ 's for different mission sizes at a given aim error and number of missions are not significant at the .05 level. For aim errors of 1 to 10 mils increasing the mission size beyond 3 shots does not change the computed value of  $I_p$  by an amount which is statistically significant.

**SUMMARY OF RESULTS**  
**EXERCISE 4**

**TABLE 1**

Aim Error S.D.	Ballistic Error S.D.	Number of Missions	Shots /Mission	I <sub>P</sub>
1.0	.5	50	3	.6523
			4	.6389
			5	.6914
1.0	.5	100	3	.6360
			4	.6381
			5	.6733
1.0	.5	200	3	.6522
			4	.6556
			5	.6789
3.0	.5	50	3	.5507
			4	.5462
			5	.5267
3.0	.5	100	3	.5691
			4	.5454
			5	.5303
3.0	.5	200	3	.5559
			4	.5419
			5	.5334
5.0	.5	50	3	.5087
			4	.5175
			5	.5190
5.0	.5	100	3	.5042
			4	.5197
			5	.5076
5.0	.5	200	3	.5067
			4	.5234
			5	.5233
10.0	.5	50	3	.3864
			4	.4633
			5	.4365
10.0	.5	100	3	.4161
			4	.4511
			5	.4341
10.0	.5	200	3	.4167
			4	.4356
			5	.4294

Exercise No. 5

It is possible to postulate different sequences of aim error which are clearly of an improving nature (i.e., which bring successive rounds closer to the target). The purpose of this exercise was to study the response of  $I_p$  to one such aiming sequence. In this exercise an initial aim error standard deviation was selected. The coordinates for the first round of a 5-round mission were drawn from a normally distributed population with the selected standard deviation.

The coordinates for subsequent rounds in the mission were drawn from distributions whose standard deviations were a fixed percentage of the value of the preceding round. This was accomplished by multiplying the initial aim error by a "reduction factor" after each draw in a mission. For example, if the initial aim error was 1 mil and the reduction factor was .9, the aim errors assumed for the 5 shots were 1, .9, .81, .729, and .6561 mils S.D., respectively.

Cases tested were 1, 3, 5 and 10 mil initial aim errors and reduction factors of .1 to 1.1 taken in .1 intervals. Mission levels were 50, 100, 200 and 400. A ballistic error of .5 mils and target 18 inches by 36 inches at 500 meters were assumed.

Figures 3 and 4 show  $I_p$  as a function of the reduction factor for the 50 mission and 400 mission cases. The smoothing effect of the additional missions is clear. It should be noted that for a one mil initial aim error, a relatively modest improvement quickly places the aim error within the bounds of the target. However, the ballistic error will cause some of these well aimed shots to miss the target. Since this effect is random and the shooter has no control over it, it causes irregular changes in the individual mission impact patterns which are assessed by  $I_p$ . When summed over a large number of missions, the variability is averaged out (as in Figure 4).

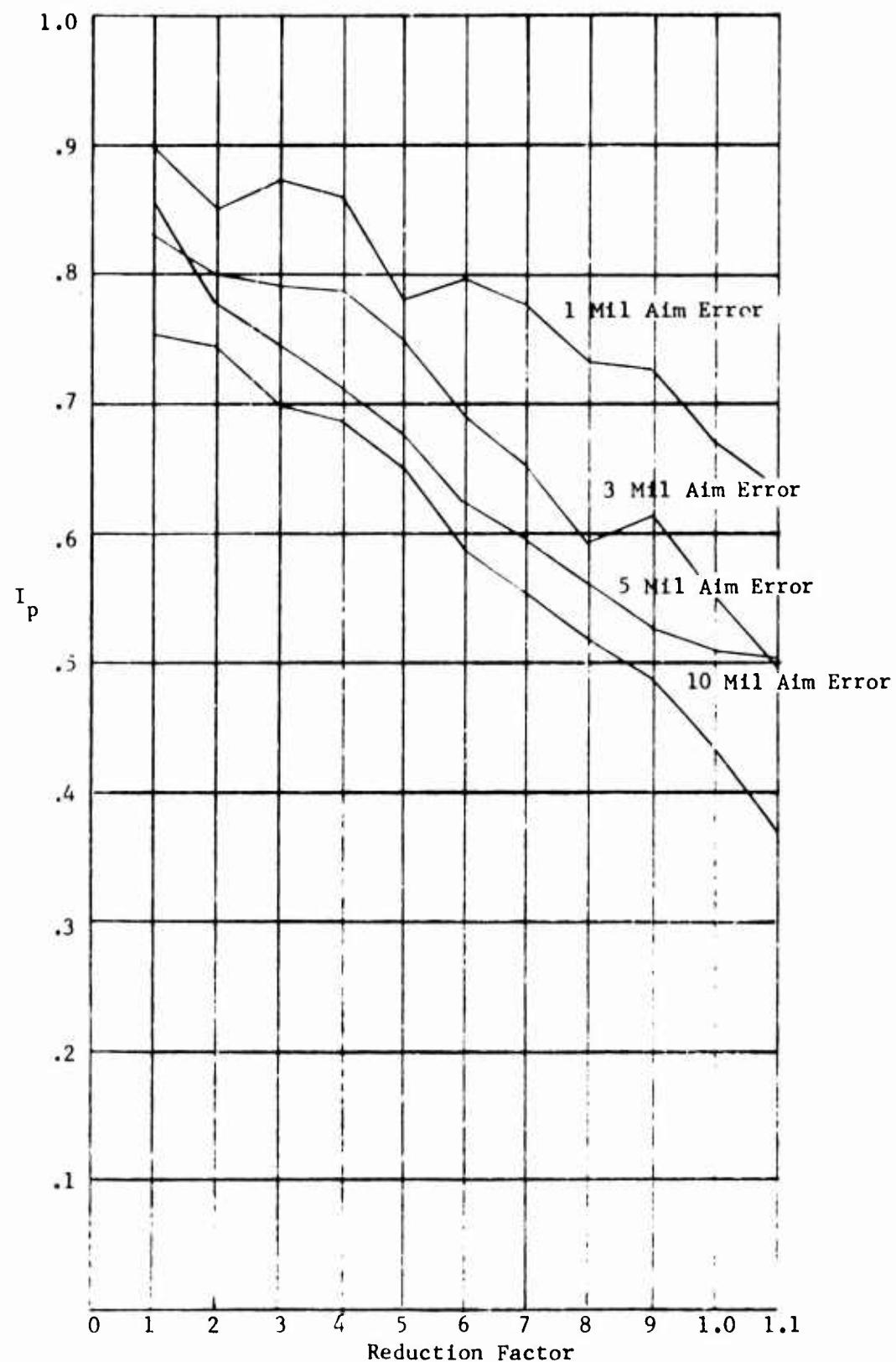


Figure 3.  $I_p$  as a Function of Reduction Factor for Four Aim Errors  
at 50 Mission Level

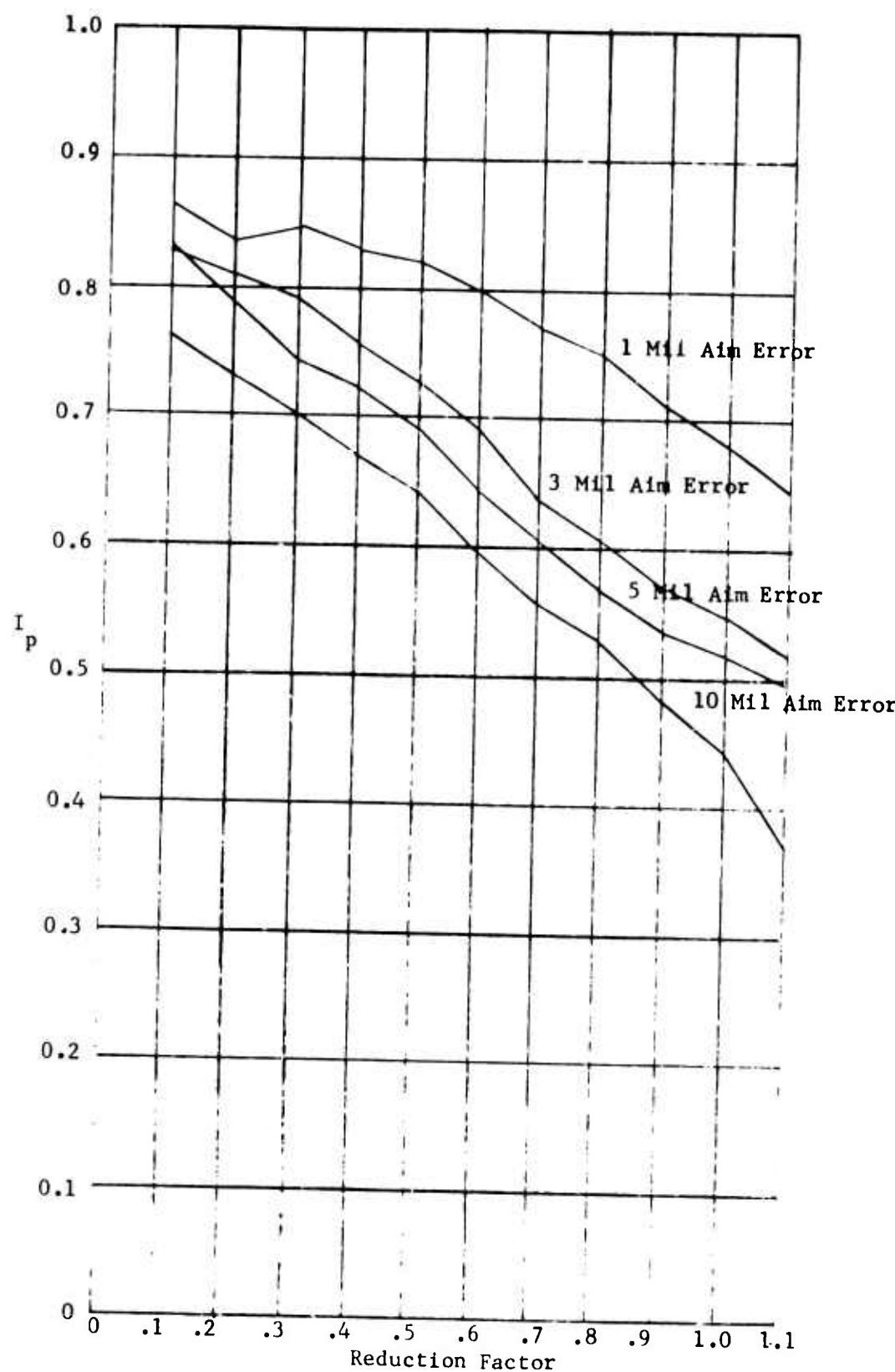


Figure 4.  $I_p$  as a Function of Reductior Factor for Four Aim Errors  
at 400 Mission Level.

Comparisons were made between the computed  $I_p$ 's for each incremental change in the reduction factor. That is,  $I_p$  for .9 reduction factor was compared with  $I_p$  for reduction factors of .8 and 1.0.

When compared at the .05 level it was found that for all aim errors at the 50 mission level the significance between  $I_p$ 's for incremental values of reduction factor was irregular. For example, the difference was significant between reduction factors of .6 and .7 but not between .5 and .6. At the 400 mission level the comparisons were statistically significant between adjacent reduction factors for all cases except the one mil initial aim error at reduction factor values of .4 or less. Hence, a general conclusion from this exercise is that for 400 missions or more the index will discriminate between two shooters who exhibit a regular pattern of aim error improvement and whose rates of improvement differ by 10% or more. However, if two shooters' initial aim errors are small and their rates of improvement great, a large number of missions may be required for the  $I_p$  to show that the differences between the two improvement rates are statistically significant.

### Exercise No. 6

If a shooter can determine the impact points of his rounds (by observing dust kicked up by the projectiles or by using tracer ammunition), his aim error for all but the first round in a given mission is likely to be dependent on the perceived impact point of his previous shot. The following sequence was used to study the response of the  $I_p$  to such a situation.

#### Aim Error

First Round: Draw from 5 mil S.D. distribution.

Second Round: Subtract the coordinates of the first round impact point (aim error + ballistic error) from the coordinates of the first round aim point. To these new coordinates add a set of coordinates drawn from a 1 mil S.D. distribution, and a set of coordinates drawn from a distribution with S.D. equal to 30% of the first round impact point coordinates. Use this final set as the aim point for round number 2.

Third Round: Repeat above procedure using the aim point and impact point of round 2.

Fourth Round: Repeat above using the aim point and impact point of round 3.

Fifth Round: Repeat above using aim point and impact point of round 4.

Ballistic error = .3, .5, .9 mils S.D.

Target Size = 18 x 36 inches at 500 meters and 6 x 12 inches at 500 meters.

The 1 mil draw and the 30% factor were an attempt to simulate the average shooter's difficulty in aiming exactly where he wants to and his inability to judge precisely the amount of aim correction that a given miss would call for.

Figure 5 shows the  $I_p$  for each target size as a function of the number of missions fired. It can be seen that for the larger target size, the index stabilizes at about 200 missions for small ballistic errors; while for the larger ballistic errors, something in excess of 400 missions is required to stabilize the index. The differences in  $I_p$  for the .3 and .5 mil ballistic errors are significant at the .05 level for 200 missions or more. The difference between .5 and .9 mil ballistic errors is not significant at 400 missions.

For the smaller target the differences in  $I_p$  are not statistically significant at the .05 level for any of the mission levels.

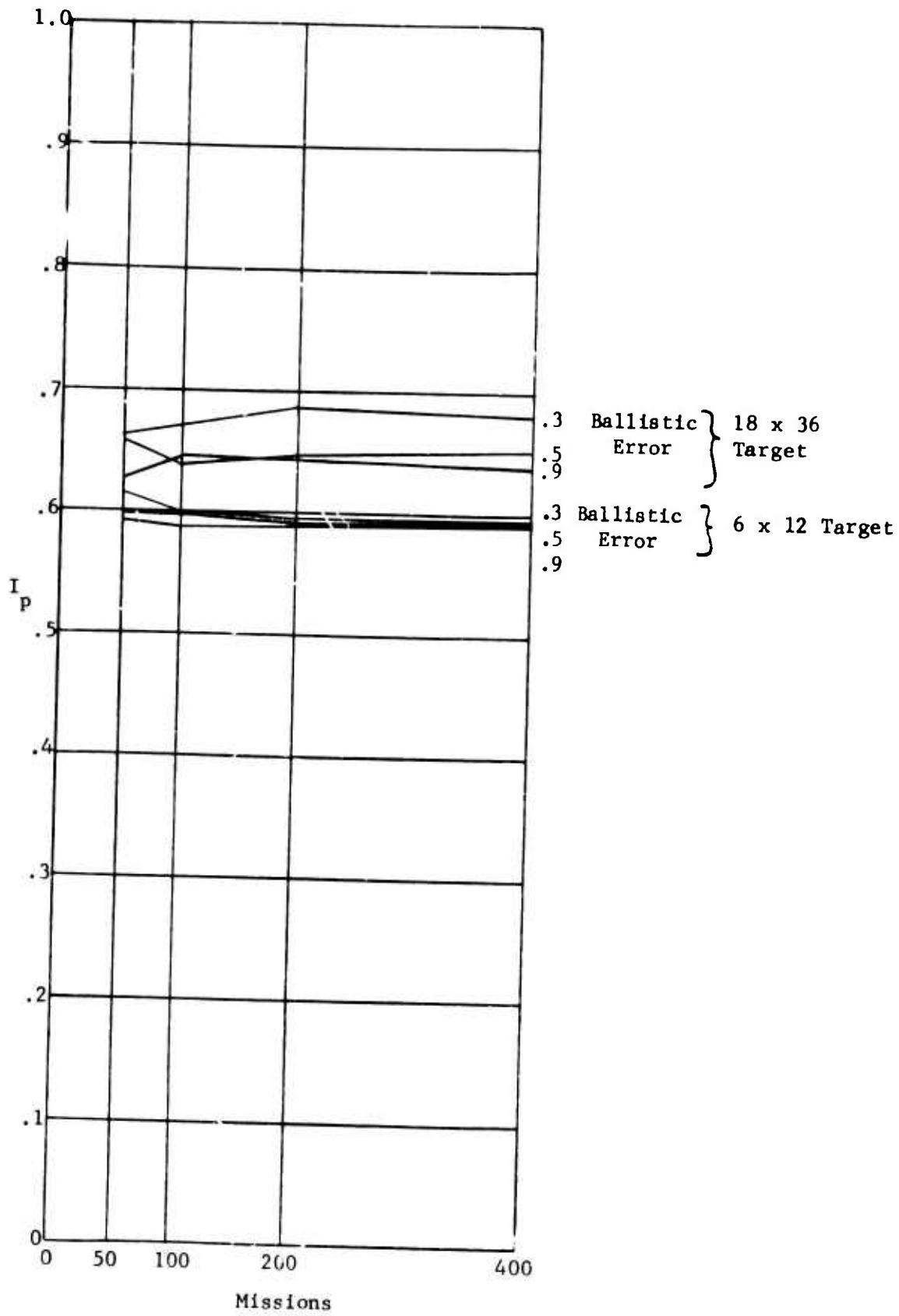


Figure 5.  $I_p$  as a Function of Number of Missions for Three Ballistic Errors and Two Target Sizes. Aim Error Defined in Exercise 6.

### Exercise No. 7

According to the rationale underlying the index of proximity, a shooter may increase the suppressive effect of his fire by purposely shooting low and by bracketing the target.<sup>1</sup> This exercise evaluated these aiming techniques by repeating Experiment No. 6 with two modifications: first, the elevation aim point for the nth round in a group was biased low by  $3/n$  mils below the target center; second, the aim point for the nth round in a group was shifted horizontally by 130 percent of the  $(n-1)$ th round impact point instead of the 100 percent shift used in the previous exercise. The following procedure was used:

#### Aim Error Vertical

First Round: Draw from the 5-mil S.D. distribution and subtract 3-mil bias.

Second Round: Subtract the ordinate of the first-round impact point (aim error + bias + ballistic error) from the ordinate of the first-round aim point. To this ordinate add an ordinate drawn from a 1-mil S.D. distribution (residual aiming error) and add another ordinate drawn from an unbiased distribution with S.D. equal to 30 percent of the first-round impact point ordinate with origin at target center and subtract a  $3/n = 1\frac{1}{2}$  mil bias. This gives the aim point for round number 2.

---

<sup>1</sup>not known to be true, but plausible.

Third Round: Repeat procedure using aim point and impact point of round 2 and letting  $3/n = 1$  mil for the bias.

Fourth Round: Repeat procedure using aim point and impact point of round 3 and letting  $3/n = 3/4$  mil for the bias.

Fifth Round: Repeat procedure using aim point and impact point of round 4 and letting  $3/n = 3/5$  mil for the bias.

#### Aim Error Horizontal

First Round: Drawing from 5-mil S.D. distribution.

Second Round: Subtract 130 percent of the first-round miss distance (aim error + ballistic error) from the first-round aim error. To this number add a number drawn from a 1-mil S.D. distribution (residual aiming error) and add another number drawn from a distribution with S.D. equal to 30 percent of the first round miss distance. This gives the aim point for round number 2.

Third Round: Repeat the procedure using the aim point and impact point of round 2.

Fourth Round: Repeat the procedure using the aim point and impact point of round 3.

Fifth Round: Repeat the procedure using the impact point and aim point of round 4.

Ballistic error = 0.3, 0.5, 0.9 mils S.D.

Target size = 18 x 36 inches at 500 meters.

Figure 6 shows  $I_p$  as a function of the number of missions. It can be seen that the index is somewhat noisy up to 200 missions. It appears to be

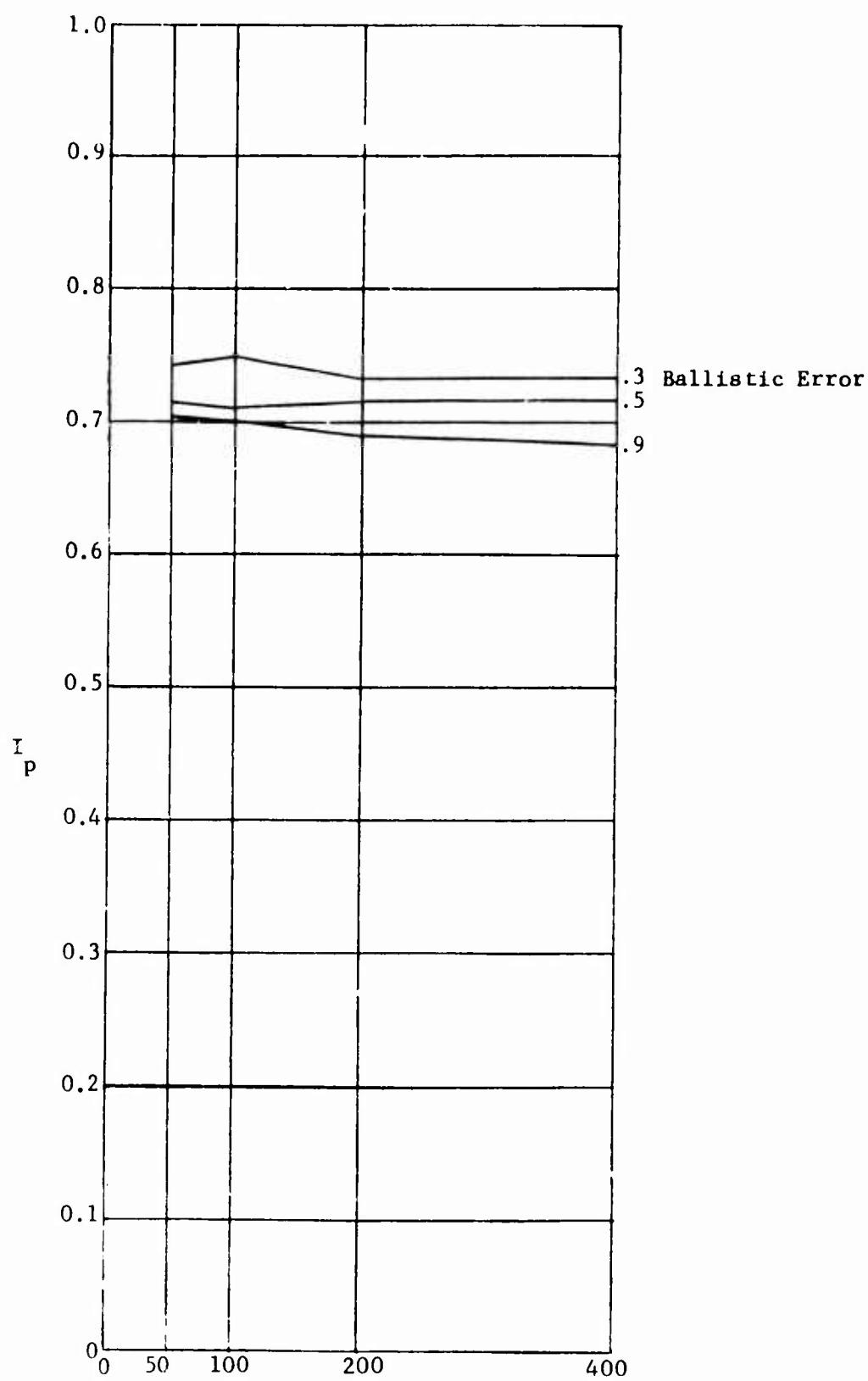


Figure 6.  $I_p$  as a Function of Number of Missions for Three Ballistic Errors. Aim Error Defined in Exercise 7.

stable at 400 missions. At the 400 mission level the differences between the  $I_p$ 's are statistically significant at the .05 level.

Figure 7 shows  $I_p$  for .5 mil ballistic error from exercises 6 and 7. Recall that in exercise 6 the simulated shooter fired in a corrective pattern which was essentially unbiased and based on the position of the previous round's impact point. In exercise 7 a similar corrective pattern was programmed; however, it was also biased so as to produce low impacts and impacts which bracketed the target. This latter pattern should produce higher  $I_p$  scores since the index is weighted to give added value to such a pattern. As shown in the graph, the computed  $I_p$  for exercise 7 were significantly greater than those for exercise 6. For comparison, the  $I_p$  for a shooter who has a 5 mil S.D. initial error and 5 mils for all subsequent shots also is shown.

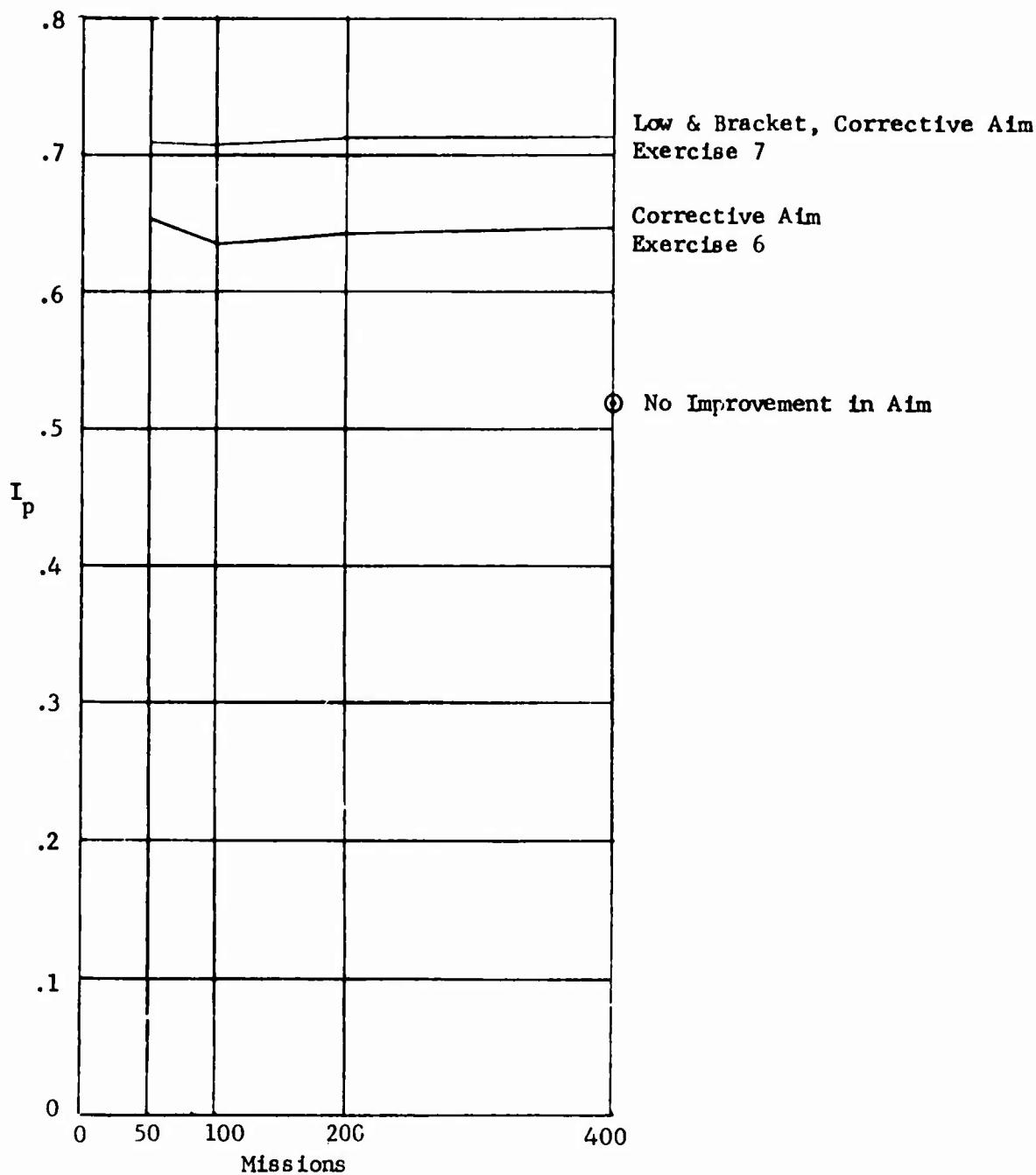


Figure 7.  $I_p$  as a Function of Number of Missions for Several Types of Aiming Patterns. Initial Aim Error 5 mils., Ballistic Error .5 Mils.

Exercise No. 8

Another function of the index of proximity is to permit the scoring of missions such as those illustrated in Figure 8. In Case A, the rifleman with a battlesight zero set at 250 meters has obtained a sight picture using holdoff and fired four rounds against the target. Having received no visual feedback to indicate where his rounds were landing, and believing his original sight picture to be correct, he has not changed his point of aim. In Case B, visual feedback is present. Although the impact point of the first round is the same as in Case A, the gunner has now changed his aim point from round to round. Yet in neither Case A nor Case B is there a hit on target.

To determine the ability of the  $I_p$  statistic to discriminate between Cases A and B (the latter being assumed to cause more target suppression), simulations were made under the following conditions:

<u>Aim Error S.D.</u>	<u>Ballistic Error S.D.</u>	<u>Target Size</u>
2 mils	.3 mil	6 x 12 inches
3 mils	.5 mil	
4 mils	.9 mil	at 500 meters
5 mils		

Figure 9 shows the results for the .5 mil ballistic error. At 400 missions the difference in  $I_p$  for the 4 and 5 mil initial aim errors and the 2 and 3 mil initial aim errors is significant at the .05 level. The difference in  $I_p$  for the 3 and 4 mil initial aim errors is not statistically significant.

FIGURE 8

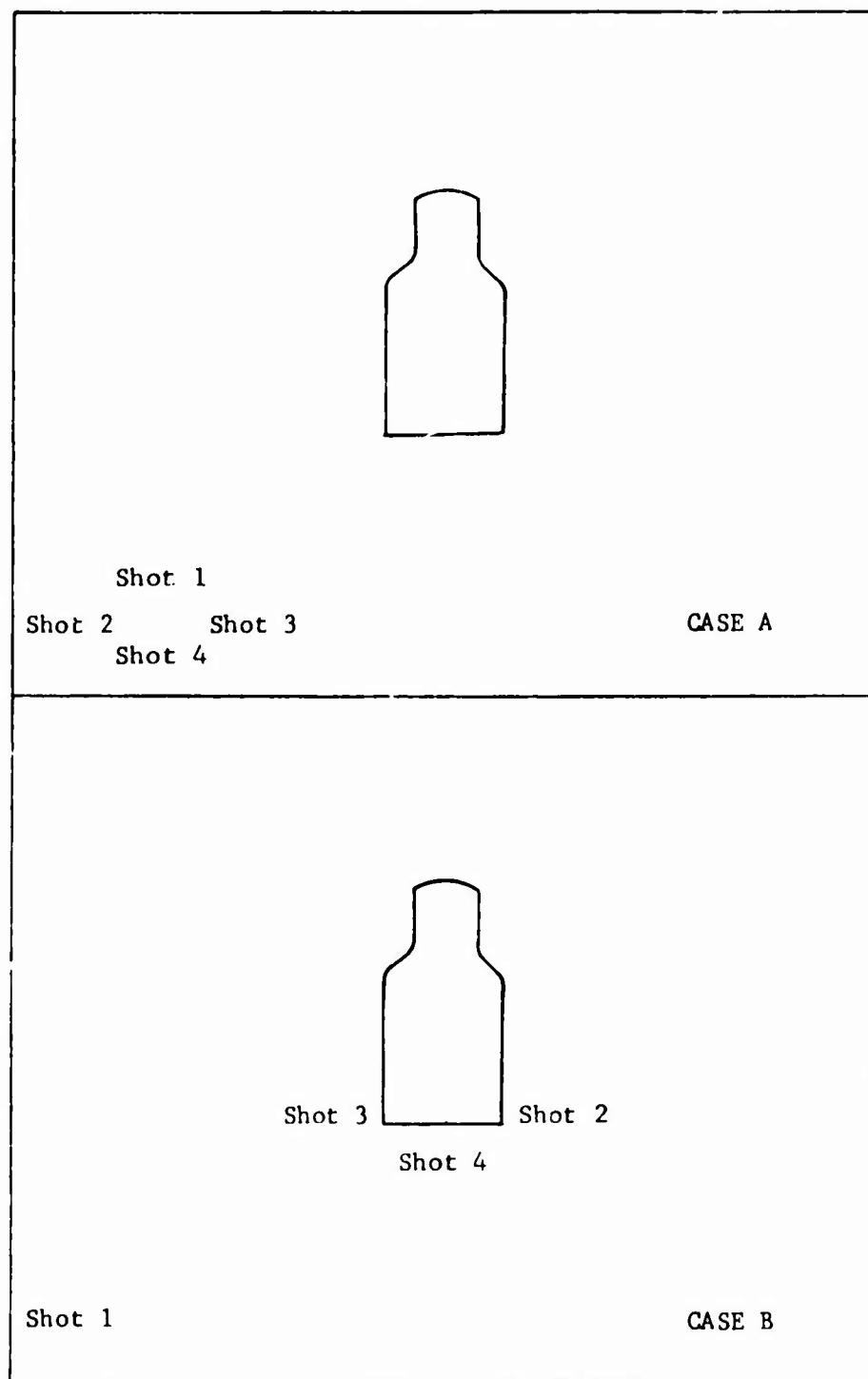


Figure 8. Comparison of Two Types of Impact Patterns.

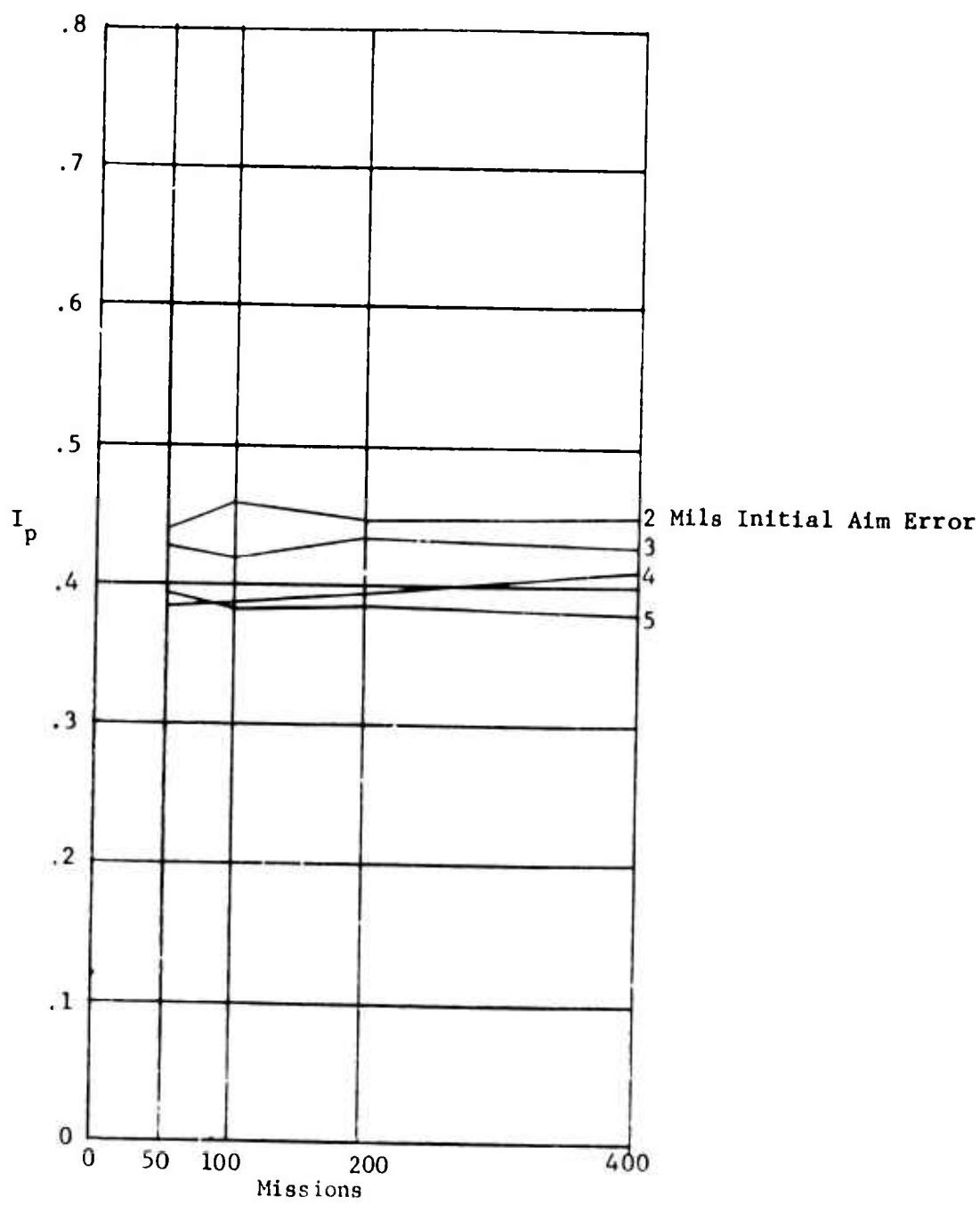


Figure 9.  $I_p$  as a Function of Number of Missions for Four Initial Aim Errors and Shot Groups as Defined in Exercise 8.

The impact patterns generated in this exercise are like those of case "A" in Figure 8. That is, in a single mission the impacts tend to be grouped about the first shot. In exercise 6, it may be recalled, each round was corrected based on the position of the previous round. Thus, the patterns generated tend to approximate case "B" of Figure 8. For the 5 mil initial aim error and .5 mil ballistic error, the computed indices were .5911 for exercise 6 and .3768 for exercise 8. The difference is significant beyond the .001 level. Thus, the index is quite sensitive to these changes in shooting patterns.

### III. COMPARISON WITH OTHER STATISTICAL MEASURES

The  $I_p$  assesses the shooter's ability to perform in certain pre-determined patterns. The statistic most frequently used to assess small arms fire is hit frequency. Figures 10, 11, and 12 show the  $I_p$  and hits/missions ratios for exercises 6, 7, and 8. It can be seen that in Exercise 6 the hit frequency stabilizes somewhat more quickly than  $I_p$  but is more sensitive to the ballistic error variable. In Exercise 7 the  $I_p$  smooths more quickly. This improvement is due to the fact that in this exercise the shooter was programmed into a pattern (low and bracketing) which the index assesses favorably. Such biasing reduces the hit frequency. In Exercise 8 the index is considerably more regular in its action than the hit frequency for the mission levels considered. This difference is due to the low number of hits and the biasing and correlation which in turn reduce the normality of the pattern.

These comparisons are not a basis for choosing between  $I_p$  and hit frequency as a measure of shooting performance, because they each measure different aspects of the shooter's impact pattern. The comparisons do give a relative indication of the efficiency of the  $I_p$  statistic as a measure of shooting performance. The  $I_p$  statistic does not discriminate as rapidly as hit frequency in those instances where the pattern is normally distributed about the center of the target. It is efficient in assessing patterns which are biased off of the target and readily responds to those shooting characteristics which have been prescribed as favorable.

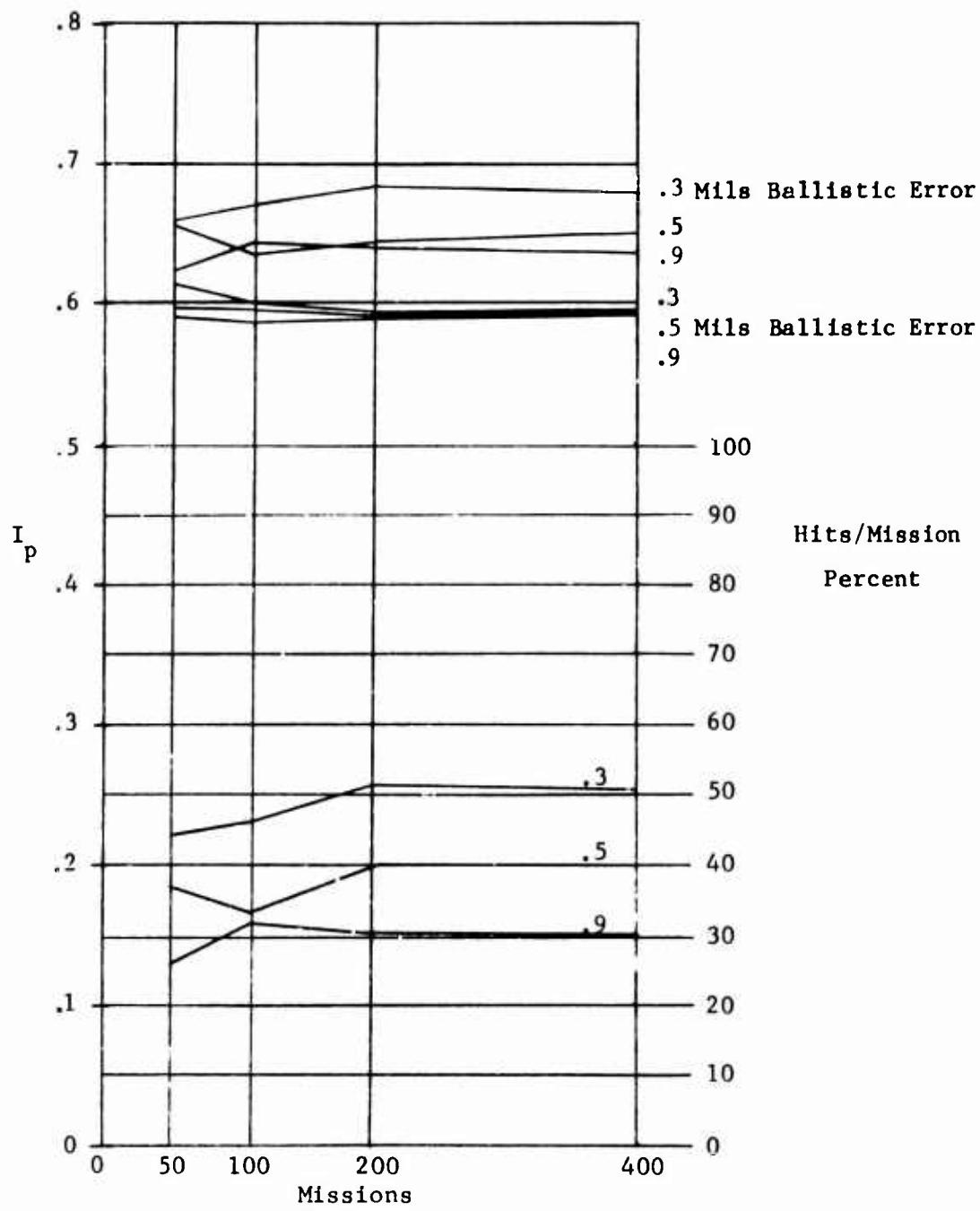


Figure 10. Comparison of  $I_p$  and Hits/Missions as a Function of Number of Missions for Exercise 6.

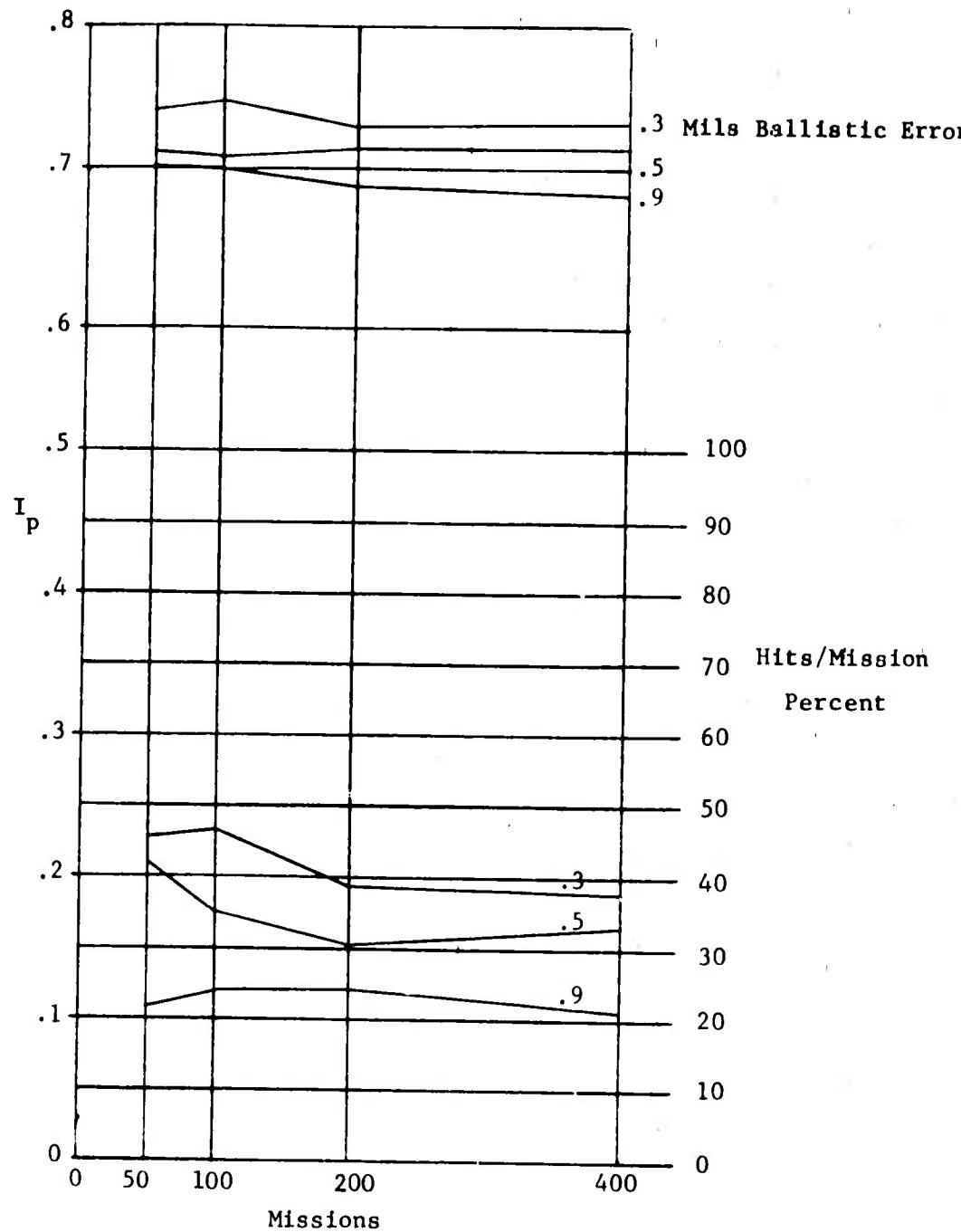


Figure 11. Comparison of  $I_p$  and Hits/Missions as a Function of Number of Missions for Exercise 7.

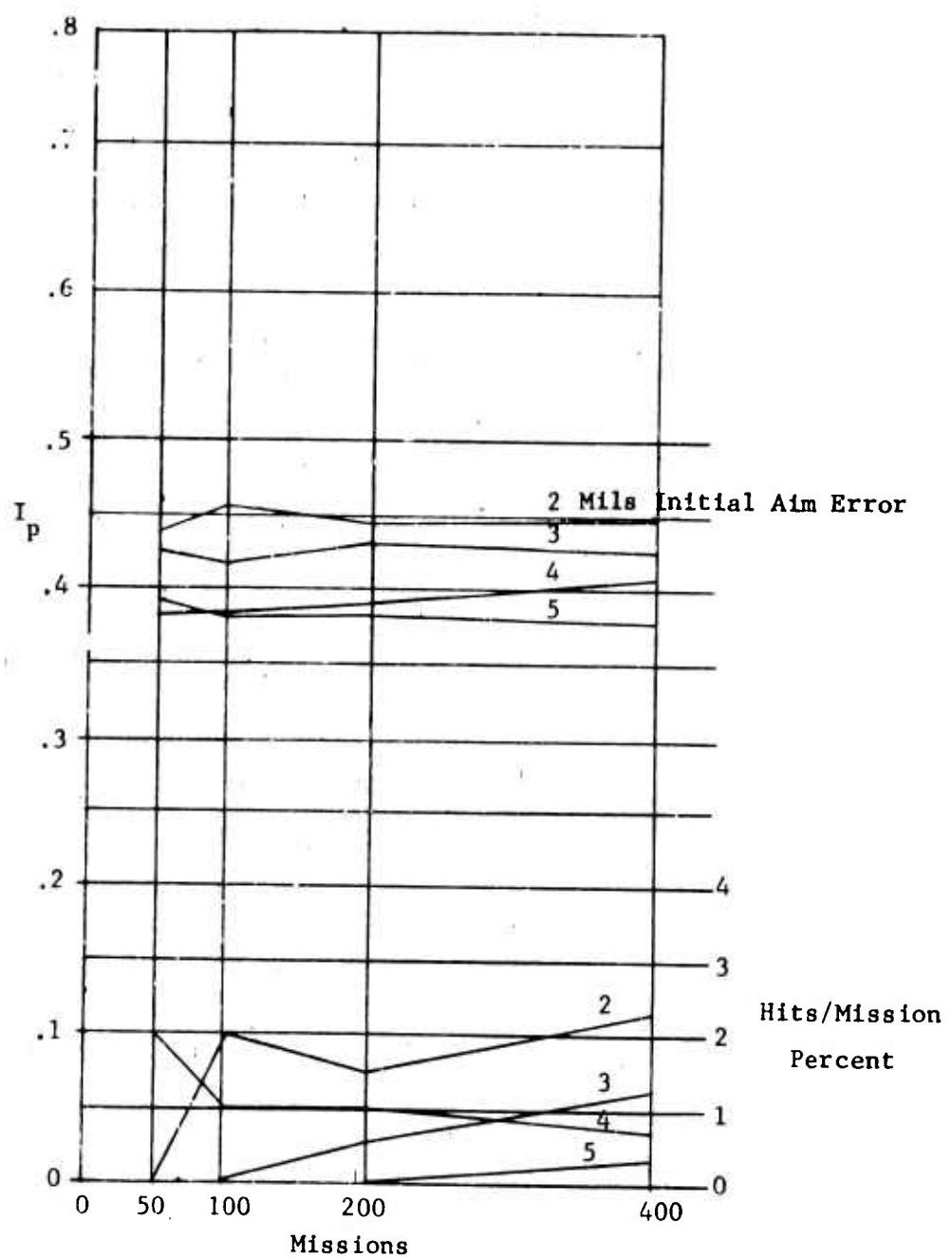


Figure 12. Comparison of  $I_p$  and Hits/Missions as a Function of Number of Missions for Exercise 8.

It may be noted that  $I_p$  uses all of the shots as inputs<sup>1</sup> while hit frequency is limited by the target size. Since all of the impact points are necessarily gathered for  $I_p$  calculations, they could also be used to compute expected hit frequency and standard deviation.

---

<sup>1</sup>(except when the first shot in the mission is a hit)

#### IV. SUMMARY AND CONCLUSIONS

A Monte Carlo computer simulation was used in eight separate exercises to evaluate the effects of changes in the magnitude of the aim error, ballistic error, and measurement error on  $I_p$ .

The first two exercises addressed the influence of impact point measurement errors on the validity of the computation of the index of proximity. The results of the analyses showed that, when summed over 50 missions or more, impact point measurement errors of the magnitude currently encountered in the field did not change the value of  $I_p$  a statistically significant amount.

Exercises three through eight were designed to study the response of  $I_p$  to various types of shooting patterns, target size, and mission size. In exercise three it was shown that  $I_p$  could be used even in situations in which the relationship between target size and aim error permits a large number of first round hits.

In exercise four it was shown that increasing the mission size beyond 3 shots did not change the computed value of  $I_p$  by a statistically significant amount.

The results of exercise five showed that for 400 missions or more the index will generally discriminate at the .05 level between two shooters who exhibit a regular pattern of aim improvement and whose rates of improvement differ by 10% or more.

Exercises six and seven simulated shooting patterns in which corrective aim points were established based on the impact point of the previous round. The index was shown to be sensitive to the corrective

patterns, and it readily reflected the low and bracketing impact patterns which are thought to be suppressive in their effect.

Exercises six and eight permitted a comparison between corrective impact patterns and ones which tend to be biased off the target. The levels of the independent variables selected for these exercises produced values of  $I_p$  which tended to stabilize around .66 and .42 respectively. The difference between these two values is statistically significant beyond the .001 level.

A comparison between  $I_p$  and hit frequency showed that the efficiency with which the index discriminates between shooting patterns is comparable to the efficiency with which hit frequency determines differences in central tendency.

V. REFERENCE

Tiedemann, A. F., Jr. and Young, R. B., Index of Proximity: A Technique for Scoring Suppressive Fire. Baltimore: AAI Corp. Engineering Report 6419, October, 1970.

## APPENDIX A

The development of the index of proximity is presented in Tiedemann and Young, 1970. The index considers five measures of performance which are thought to contribute to the suppressive effectiveness of small arms fire. These measures are the relative positions of sequential rounds, the miss distance of the round closest to the target, the rate of closure on the target, whether the round strikes above or below the target, and whether alternating rounds bracket the target.

The equation defining the index of proximity is given on page 38. The reader is referred to the referenced report for a detailed explanation of its development and use.

Preceding page blank

$$I_p = \frac{j}{n} \left\{ \left( \frac{h}{n-1} \right) w_1 + \left( \frac{m}{n} \right) w_4 + \left( \frac{k}{n-1} \right) w_5 \right\} + \left( \frac{r_p - r_{min}}{r_p} \right) w_2 + \frac{2}{n} \left( \frac{r_p - r_{min}}{r_p} \right) w_3$$

subject to  $n \geq 2$

$$w_1 + w_2 + w_3 + w_4 + w_5 = 1$$

Where:

$I_p$  = index of proximity

$h$  = number of sequentially closer rounds

$j$  = number of impact points within the limit circle

$k$  = number of alternating strikes or over corrections

$m$  = number of low rounds

$n$  = number of rounds/fired mission

$r_p$  = radius of allowable miss circle

$r_{min}$  = radius from the target to the first shot which impacts within the limit circle

$w_1$  = weighting factor for sequentially closer rounds

$w_2$  = weighting factor for closest round

$w_3$  = weighting factor for rate of closure

$w_4$  = weighting factor for low rounds

$w_5$  = weighting factor for bracketing the target

The value of  $I_p$  thus calculated will always lie on the interval  $[0,1]$ . A quantitative measure for any mission of any number of rounds can be calculated where  $n \geq 2$  and the first round is not a hit.